Quantum Spin Liquids

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Quantum non-locality

EPR

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

??where is the information??
Schrödinger’s Cat

UNSTABLE to decoherence - uncontrolled entanglement with the environment
Strange Stuff

Phil Anderson, 1973

a “quantum liquid” of spins

\[ \Psi = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

Resonating Valence Bond state
Strange Stuff

Phil Anderson, 1973

a “quantum liquid” of spins

\[ \Psi = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

Resonating Valence Bond state
Ordinary (local) Matter

We can consistently assign local properties (elastic moduli, etc.) and obtain all large-scale properties:

- Measurements far away do not affect one another
- From local measurements we can deduce the global state
Ordinary (local) Matter

Hamiltonian is local

\[ H = \sum_x \mathcal{H}(x) \]

\( \mathcal{H}(x) \) has local support near \( x \)

Ground state is “essentially” a product state

\[ |\Psi\rangle = \bigotimes_A |\psi\rangle_A \]

no entanglement between blocks
“Essentially” a product state?

• Adiabatic continuity

\[ |\Psi\rangle = \bigotimes_A |\psi\rangle_A \]

n.b. This is not true for gapless fermi systems
“Essentially” a product state?

- Entanglement scaling

\[ \rho_A = \text{Tr}_\bar{A} |\Psi\rangle \langle \Psi| \]

\[ S(A) = -\text{Tr}_A (\rho_A \ln \rho_A) \]

\[ S(A) \sim \sigma L^{d-1} \quad \text{area law} \]

satisfied with exponentially small corrections
Best example: ordered magnet

Hamiltonian

\[ H = \sum_{(ij)} J_{ij} S_i \cdot S_j \]

exchange is short-range: local

ordered state

\[ |\Psi\rangle \approx \bigotimes_i |S_i \cdot \hat{n}_i = +S\rangle \]

block is a single spin
Quasiparticles

- Excited states ~ excited levels of one block

- Local excitation can be created with operators in one block
- Localized excitation has discrete spectrum with non-zero gap, and plane wave forms sharp band
- Quantum numbers consistent with finite system: no emergent or fractional quantum numbers
Spin wave

\[ \omega(k) \approx \Delta - 2t \cos k_x a - \cdots \]

\[ |f\rangle = S_k^+ |i\rangle \]

Line shape in Rb$_2$MnF$_4$
Quantum spin liquid

Entanglement -> non-local excitation

$\Psi = \Psi + \Psi + \cdots
\Psi = \Psi_{\text{spinon}} + \cdots$

"quasiparticle" above a non-zero gap
Fractional quantum number

excitation with $\Delta S = 1/2$
not possible for any finite
cluster of spins
always created in pairs by any
local operator
No spin waves

- Magnon is not elementary: decays into two spinons

\[ K-k, \Omega - \omega \]
\[ k, \omega \quad \text{magnon } S=1 \]

\[ k-k', \omega - \omega' \quad \text{spinon } S=1/2 \]

\[ k', \omega' \]

- Sharp peaks should be reduced or absent in the spin structure factor

\[ \omega = \epsilon(k') + \epsilon(k-k') \]
c.f. One dimension

A. Tennant et al, 2001

KCuF$_3$
Anyons

ψ → −ψ  "mutual semions"
Topological phases

Anderson’s RVB state is thus an example of a “topological phase” - the best understood sort of QSL

Understood and classified by anyons and their braiding rules in 2d
Robustness arises from topology: a QSL is a stable phase of matter (at T=0)
Quantum spin liquid

\[ \Psi = \Psi_1 + \Psi_2 + \ldots \]

For ~500 spins, there are more amplitudes than there are atoms in the visible universe!

Different choices of amplitudes can realize different QSL phases of matter.
Construct QSL state from free fermi gas with spin, with 1 fermion per site (S=0)

\[ |\Psi_0\rangle = \prod_{k \in FS} c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger |0\rangle \]

"partons"
"spinons"
Gutzwiller Construction

- Project out any components with empty or doubly occupied sites

\[ |\Psi\rangle = \hat{P}_G |\Psi_0\rangle \]

\[ = c_1 + c_2 + c_3 + \cdots \]

“partons”
“spinons”
Gutzwiller Construction

• Can build many QSL states by choosing different free fermion states

\[ |\Psi\rangle = \hat{P}_G |\Psi_0\rangle \]

\[ = c_1 + c_2 + c_3 + \cdots \]

"partons"
"spinons"
Classes of QSLs

- Topological QSLs
- $U(1)$ QSL
- Dirac QSLs
- Spinon Fermi surface
Classes of QSLs

- Topological QSLs
- U(1) QSL
- Dirac QSLs
- Spinon Fermi surface

- anyonic spinons
- electric+magnetic monopoles, photon
- strongly interacting Dirac fermions
- non-Fermi liquid “spin metal”
Strange stuff

where do we find it?
Kagomé antiferromagnet

\[ H = J \sum_{\langle ij \rangle} S_i \cdot S_j + \ldots \]

Very large classical degeneracy

likely to be a QSL

V. Elser, 1989 + many many others
S=1/2 kagomé AF

- Rather definitive evidence for QSL by DMRG

S. Yan et al, 2010

many other studies support existence of some QSL phase
Theory

• What kind of QSL?

S. Depenbrock et al, 2012

gapped, topological QSL

gapless Dirac QSL

+ various other proposals with weaker quantitative support

Y. Ran et al, 2007
F. Becca...
Y.C.He et al, 2016
Triangular lattice w/ ring exchange

- Motrunich (2005): ring exchange stabilizes a spin liquid
Triangular lattice w/ ring exchange

• Motrunich (2005): ring exchange stabilizes a spin liquid

• Motrunich, Lee/Lee: spin liquid state favored by ring exchange is the "spinon Fermi sea" state
SOC triangular

Heavy elements:
highly localized electrons, strong spin-orbit coupling

\[ H = \sum_{\langle ij \rangle} \left[ J_\pm (S_i^+ S_j^- + S_i^- S_j^+) + J_z S_i^z S_j^z \right. \]

\[ + J_{\pm \pm} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-) \]

\[ + iJ_{\pm z} (\gamma_{ij}^* S_i^z S_j^+ - \gamma_{ij} S_i^z S_j^- + (i \leftrightarrow j)) \]\n
XXZ bond-dependent couplings

Y. Li et al, 2015
SOC triangular

QSLs versus magnetic order

![Diagram showing the classical and quantum phase diagrams for the spin system. The classical phase diagram is on the left, and the quantum phase diagram is on the right. The parameters $J_3/J_2$ and $J_2/J_\pm$ are used to define the phases, including 120° AFM, Incommensurate $(q,0)$, Stripe Order, Incommensurate $(0,q)$, and 120° AFM phases.]

Some window exists for Dirac QSL

Kitaev model

Kitaev’s honeycomb model

\[ H = \sum_{i, \mu} K_{\mu} \sigma_{i}^{\mu} \sigma_{i+\mu}^{\mu} \]

1. The model

Spin \( \frac{1}{2} \) on each site.

exact parton construction

- spinon is a gapless Majorana fermion
How to probe QSLs?

Two main characteristics:

• **Massive entanglement**
  - Almost no experiments known to probe this.

• **Fractional/non-local excitations**
  - Probed by most low energy response measurements. Challenge is to distinguish the fractional/non-local nature.
A rough guide to experiments on QSLs

Does it order?
- NMR line splitting
- muSR oscillation
- thermodynamic transition via specific heat, susceptibility
- Bragg peak in neutron/x-ray

Is there a gap?
- Specific heat
- NMR $1/T_1$
- Dynamic susceptibility
- T-dependence of $\chi$

Exotica
- Local measurements
- thermal Hall
- ARPES (on insulator!)
- Proximity effects

Delocalized excitations?
- thermal conductivity
- INS

Structure of excitations?
- E(k) from INS,RIXS
- optics, Raman
Scattering

Herbertsmithite

Heisenberg-like with $J \sim 200\text{K}$

no order down to $50\text{mK}$

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Helton et al, 2007
Scattering

T-H Han et al, 2012

continuum scattering expected
...but probably with more structure?

VS

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Spin flip produces a free Majorana fermion and two immobile fluxes

\[ \sigma_i^z = \begin{array}{c} i \\ i \end{array} \]

J. Knolle et al, 2014
Xueyang Song, Yi-Zhuang You + LB, 2016

dynamical spin correlations in the Kitaev QSL

FIG. 4. The spectral function along high symmetry line in the Brillouin zone at the isotropic point. The simplest form included in the lattice model.

FIG. 5. The spectral function versus frequency at \( \omega/\Gamma \).

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Kitaev QSL

This work illustrates that the generic low-frequency spin correlations can be written down the expression for the dynamical structure factor as listed in eq. (9) for small frequency therefore are linear/cubic in any perturbation that doesn't induce phase transition. General dynamical spin correlations in the Kitaev QSL fluctuates in space with \( q \rightarrow 0 \) and Fig. 6, respectively. It's also clear that product \( \sigma_s \sigma_s \) or with gradients in between will contribute or with gradients in between will contribute higher powers of frequency to the low-energy weight.

Since \( [\sigma_s, \sigma_s] = 0 \) doesn't possess non-vanishing zero-order field product term after series expansion of the field \( \sigma_s \sigma_s, \sigma_s \sigma_s \) the power law relation can be deduced in terms of single-particle green function in reciprocal space. The generic low-frequency spin correlations can be written down the expression for the dynamical structure factor as listed in eq. (9) for small frequency therefore are linear/cubic in any perturbation that doesn't induce phase transition. General dynamical spin correlations in the Kitaev QSL fluctuates in space with \( q \rightarrow 0 \) and Fig. 6, respectively. It's also clear that product \( \sigma_s \sigma_s \) or with gradients in between will contribute or with gradients in between will contribute higher powers of frequency to the low-energy weight.

The spectral function vanishes for \( \omega/\Gamma \) higher powers from previous dimensional analysis. It's also clear that product \( \sigma_s \sigma_s \) or with gradients in between will contribute higher powers of frequency to the low-energy weight. General dynamical spin correlations in the Kitaev QSL fluctuates in space with \( q \rightarrow 0 \) and Fig. 6, respectively. It's also clear that product \( \sigma_s \sigma_s \) or with gradients in between will contribute higher powers of frequency to the low-energy weight.
alpha-RuCl₃

"column" of scattering suggested to be related to Majorana spinons

A. Banerjee et al, 2017

S.-H. Do et al, 2017

c.f.
Low energy signatures: triangular organics

- Molecular materials which behave as effective triangular lattice $S=1/2$ antiferromagnets with $J \sim 250K$
- Significant charge fluctuations
La lattice Heisenberg model with an AFM interaction energy. At ambient pressure, the network of interdimer transfer integrals forms a Mott insulator at ambient pressure and becomes a metal repulsion inhibits the hole transfer [6]. In fact, it is the only spin-liquid system to exhibit the Mott transition, to new area of research.

Metallic and insulating spin-liquid phases are an interesting to 32 mK. These results suggested the spin-liquid state at 0.6 0.7 0.8 0.9 1.0

(1/T_1 T)_{\text{max}}$

$T = \text{const.}$

$R = R_0 + AT^2$

(T_1 T)^{\text{max}}$

Superconductor

Mott insulator

(Spin liquid)

Metal

onset $T_C$

Crossover

30

40

50

60

70

0.6 0.7 0.8 0.9 1.0

$T' / T$

$T (K)$

$T (K)$

$T (K)$

Pressure (10^{-1} \text{GPa})

$k$-(ET)$_2$Cu$_2$(CN)$_3$

K. Kanoda group (2003-)

$\beta'$-Pd(dmit)$_2$

R. Kato group (2008-)
Evidence for lack of static moments: $f > 1000!$
Specific Heat

- \( C \sim \gamma T \) indicates gapless behavior with constant density of states

\[
\kappa-(ET)_2Cu_2(CN)_3
\]

\( \gamma_{Cu} \sim 0.7 \)

S. Yamashita et al, 2008

\( \beta'-Pd(dmit)_2 \)
Thermal conductivity

- Huge linear thermal conductivity indicates the gapless excitations are propagating, at least in dmit.

- Estimate for a metal would correspond to a mean free path $l \sim 1 \mu m \approx 1000 a$ !

M. Yamashita et al, 2010
Thermal conductivity

M. Yamashita et al, 2010
Spinon Fermi surface

- How could we firm this up?
- Spinons should be confined to 2d. Can we see evidence of this?
  - e.g. $k_c < k_{ab}$ (c.f. Y. Werman et al, 2017)
- See signs of $k_F$?
  - Quantum oscillations, RKKY
- Possible quantization effects in small systems
- Observe conversion of spinons to electrons in adjacent metal (c.f. T. Senthil, 2008)
Spinons → Electrons?

T. Furukawa et al, 2017
Other QSLs in organics?

- Topological QSLs
- U(1) QSL
- Dirac QSLs
- Spinon Fermi surface

- anyonic spinons
- electric+magnetic monopoles, photon
- strongly interacting Dirac fermions
- non-Fermi liquid “spin metal”
Thanks for your attention

References here:
https://spinsandelectrons.com/
https://spinsandelectrons.com/pedagogy/