A strongly correlated metal from coupled SYK models

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Transport

\[ j = \sigma E \quad j_e = -\kappa \nabla T \]

• Arguably most important aspect of quantum materials: electrical and thermal conductivity (and crossed coefficients)

• Sensitive, versatile

• Probes extreme long wavelength, low frequency
Theory

• *Understanding* of transport mainly through electron *quasiparticle* picture

• Boltzmann equation:

\[
\left[ \partial_t + v_n(k) \cdot \nabla_r - eE \cdot \nabla_k \right] f_n = \left. \frac{\partial f_n}{\partial t} \right|_{\text{collision}}
\]

Linearizing this around equilibrium gives conductivities in terms of band velocities and scattering rates
Ultra-quantum transport

• How does transport work when quasiparticles are not adiabatically connected to electrons?

• Or when quasiparticles scatter very strongly?

• Or if there are no quasiparticles at all?
Convergence of ideas and experiments

- Experiments on
  - non-Fermi liquid metals
  - ultra-clean semi-metals
  - thermal conductivity in quantum magnets
  - electron spin resonance of spin liquids

- Theoretical approaches
  - Gauge/gravity duality
  - SYK model and related large N theories
  - Quantum hydrodynamics
  - Field theory
This Talk

• Strongly correlated metals from SYK models - a route to calculable non-quasiparticle transport
Fermi Liquid Theory

Landau provided justification for quasiparticle picture in metals when $T \ll E_F$

Low energy excitations act like electrons and holes but with wavefunction dressing ($Z<1$), effective mass, and Landau interactions

$$E = \sum_k \epsilon_k \delta n_k + \frac{1}{2V} \sum_{k,k'} U_{k,k'} \delta n_k \delta n_{k'}$$

scattering is weak because not so many low energy qp states to scatter to
Heavy Fermi Liquids

\[ C \sim \gamma T \]
\[ \rho(T) - \rho(0) \sim AT^2 \]

Both \( \gamma \) and \( A \) huge

Behave like Fermi liquid with tiny \( E_F \) and large electron mass, but only for \( T << E_F \)
Non-Fermi Liquids

T-linear resistivity:

- Many materials
- Often nearby to unconventional superconductivity
- Symptom of a different type of metal?
- If no quasiparticles exist, what is the starting point?

\[ \frac{1}{\tau} \sim T \]
Sachdev-Ye-Kitaev model

A toy exactly soluble model of a non-Fermi liquid

\[ H = \sum_{i<j,k<l} U_{ijkl} c_i \dagger c_j \dagger c_k c_l \]

\[ |U_{ijkl}|^2 = \frac{2U^2}{N^3} \]

Like a strongly interacting quantum dot or atom with complicated Kanamori interactions between many "orbitals"
SYK Model

Sachdev-Ye, 1993: Model has a soluble large-N limit

\[ \Sigma = \frac{1}{\omega_n - \Sigma(i\omega_n)} + O(1/N) \]

In equations: very similar to DMFT:

\[ G(i\omega_n) = \frac{1}{i\omega_n - \Sigma(i\omega_n)} \]
\[ \Sigma(\tau) = -U^2 G(\tau)^2 G(-\tau) \]

Solution:

\[ G(i\omega) \sim \frac{1}{\sqrt{\omega}} \]

not a pole: non-Fermi liquid
SYK Model

Why not quasiparticles?

Georges, Parcollet, Sachdev, 2001: **ground state entropy!**

\[
\ln(2) = 0.69...
\]

\[
s_0 = 0.46...
\]

(at half-filling)

Many states available for scattering

“level spacing” \( \sim U \exp(-Ns_0) \)
Density dependence

\[ H \to H - \mu N \]

\[ \mathcal{N} = \sum_i c_i^\dagger c_i \]

\[ Q = \frac{\mathcal{N}}{N} - \frac{1}{2} \]

![Entropy](image)

![Energy](image)

• Compressibility is constant at \( T=0 \)

\[ K = \left. \frac{\partial Q}{\partial \mu} \right|_{\mu=0} = \frac{1.04}{U} \]

 curvature = \( 1/K \)
SYK Summary

- Compressible
- Ground state entropy
- Non-Fermi liquid

\[ K(T = 0) = \frac{1.04}{U} \]

\[ S(T = 0)/N = 0.46 \ldots \]

\[ G(i\omega) \sim 1/\sqrt{\omega} \]
SYK Summary

- Compressible
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\[ K(T = 0) = \frac{1.04}{U} \]
\[ S(T = 0)/N = .46 \ldots \]
\[ G(i\omega) \sim 1/\sqrt{\omega} \]

Chaos

\[ \langle [\mathcal{O}(0), \tilde{\mathcal{O}}(t)]^2 \rangle \sim \frac{1}{N} e^{\lambda_L t} \]
\[ \lambda_L = \frac{2\pi k_B T}{\hbar} \]

Holography

- black holes
- chaos and the Regge limit
- AdS\(_3\)
- strange metals

*circa 2015* slide from D. Stanford, IAS, 2017
Building a metal

\[ H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{lx} + \sum_{\langle xx'\rangle} \sum_{i,j} t_{ij,xx'} c_{i,x}^\dagger c_{j,x'} \]

\[ |t_{ij,xx'}|^2 = t_0^2/N. \]
Building a metal

\[ H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ijkl,xx'} c_{ix}^\dagger c_{jx}^\dagger c_{k,x'} c_{l,x'} + \text{h.c.} \]

Omitting relevant 1-electron hopping leaves system NFL even at T=0

Other work: 2-electron hopping

Y. Gu et al, arXiv:1609.07832
R. Davison et al, arXiv:1612.00849
Building a metal

\[ H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c_{i,x}^\dagger c_{j,x}^\dagger c_{k,x} c_{l,x} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c_{i,x}^\dagger c_{j,x} \]

competition!

\( t/U \ll 1 \) interesting
Building a metal

\[ H = \sum_{x} \sum_{i<j,k<l} U_{ijkl,x} c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c_{i,x}^\dagger c_{j,x'} \]

NFL \quad FL
Self-consistent equations

\[ G(i\omega_n)^{-1} = \frac{1}{i\omega_n + \mu - \Sigma_4(i\omega_n) - z t_0^2 G(i\omega_n)}, \]
\[ \Sigma_4(\tau) = -U_0^2 G(\tau)G(-\tau), \]

Strong similarities to DMFT equations

Mathematical structure appeared in early study of doped t-J model with double large N and infinite dimension limits: O. Parcollet+A. Georges, 1999
**Coherence scale**

\[
G(i\omega_n)^{-1} = i\omega_n + \mu - \Sigma_4(i\omega_n) - zt_0^2 G(i\omega_n),
\]
\[
\Sigma_4(\tau) = -U_0^2 G(\tau)^2 G(-\tau),
\]

Rescaling
\[
\tilde{\omega} = \frac{\omega}{\tilde{E}_c}, \quad \tilde{\tau} = \tau \tilde{E}_c, \quad \tilde{G}(i\tilde{\omega}) = \frac{i}{\tilde{U}} \tilde{G}(\tilde{\omega}), \quad \tilde{\Sigma}(i\tilde{\omega}) = \frac{1}{\tilde{t}} \Sigma(i\omega)
\]
\[
\tilde{t} = \left(\frac{\tilde{z}}{2}\right)^2 \frac{1}{2} \tilde{t}
\]

\[
\tilde{G}(i\tilde{\omega}) = \frac{\tilde{t}}{\tilde{U}} i\tilde{\omega} - \tilde{\Sigma}(i\tilde{\omega})
\]

\[
\tilde{\Sigma}(\tilde{\tau}) = -\tilde{G}(\tilde{\tau})^2 \tilde{G}(-\tilde{\tau}) + 2\tilde{G}(\tilde{\tau}),
\]

For \(t \ll U\), a single universal coherence scale appears

\[
\tilde{E}_c = \frac{\tilde{t}^2}{\tilde{U}}
\]
Coherence scale

We solve these equations in a real time Keldysh formulation numerically and determine asymptotics analytically.

Narrow “coherence peak” appears in spectral function: heavy quasiparticles form for $T \ll E_c$

Quasiparticle weight $Z \sim t/U$
Entropy

Level repulsion: entropy is released for $T<E_c$!

Universal scaling forms

$$\frac{S}{N} = S(T/E_c)$$
$$\frac{C}{N} = T/E_c S'(T/E_c)$$

$$\gamma \equiv \lim_{T \to 0} \frac{C}{T} = \frac{S'(0)}{E_c}$$

Sommerfeld enhancement

$$m^*/m \sim U/t$$
Compressibility

For $t \ll U$, compressibility is almost unaffected by hopping

$$K = \frac{\partial Q}{\partial \mu} \bigg|_{\mu=0} = \frac{1.04}{U} \ll \gamma \sim \frac{U}{t^2}$$

??How to reconcile with Sommerfeld enhancement??

Free fermions

NFL $\longrightarrow$ FL

$K = g(\epsilon_F)$

$$\gamma = \frac{\pi^2}{3} g(\epsilon_F)$$
Landau Fermi Liquid

• Landau Fermi Liquid Theory

\[ \delta E = \sum_a \epsilon_a \delta n_a + \frac{1}{2} \sum_{a,b} f_{ab} \delta n_a \delta n_b \]

• Compressibility is renormalized by Fermi liquid parameter \( F = g(E_F) f \)

\[ \frac{\gamma}{K} = \frac{\pi^2}{3} (1 + F) \]

\[ \frac{\gamma}{K} \sim \left( \frac{U}{t} \right)^2 \]

\( F \sim \left( \frac{U}{t} \right)^2 \gg 1 \)
Transport

Quasiparticle picture applies only for $T \ll E_c$

More generally, we use hydrodynamics

Einstein-Smoluchowski relation

$$\sigma = e^2 \frac{\partial n}{\partial \mu} D$$
Transport

Quasiparticle picture applies only for $T \ll E_c$

More generally, we use hydrodynamics

$$\sigma = \lim_{\omega \to 0} \lim_{p \to 0} \frac{-i\omega}{p^2} D_{Rn}(p, \omega)$$

✦ Calculate density response using Keldysh method.
✦ Do analogously for thermal conductivity
✦ Very convenient collective field formulation - fully non-perturbative calculations possible

N.B. Because of randomness, momentum is not a hydrodynamic variable
Transport

Quasiparticle picture applies only for $T \ll E_c$

More generally, we use hydrodynamics

Effective action:

\[
\begin{align*}
G_{x,ss'}(t, t') &\rightarrow G_{x,ss'}(t - t') e^{-i(\varphi_s(x,t) - \varphi_{s'}(x,t'))} \\
\Sigma_{x,ss'}(t, t') &\rightarrow \Sigma_{x,ss'}(t - t') e^{-i(\varphi_s(x,t) - \varphi_{s'}(x,t'))},
\end{align*}
\]

known from SYK solution

\[
iS_\varphi = -2K \sum_p \int_{-\infty}^{+\infty} d\omega \varphi_{c,\omega} (i\omega^2 - D_\varphi p^2 \omega) \varphi_{q,-\omega}.
\]
Transport

Generalized resistivity

\[ \rho_c = \frac{1}{\sigma} \quad \rho_e = \frac{T}{\kappa} \]

scaling

\[ \rho_\zeta(t_0, T \ll U_0) = \frac{1}{N} R_\zeta \left( \frac{T}{E_c} \right) \]

Fermi liquid

\( R = R_0 + A T^2 \)

for \( T \ll E_c \)

Crossover from heavy FL to strange metal

Linear in \( T \) for \( E_c \ll T \ll U \)
Incoherent tunneling

\[ J_{x,x'} \sim it \left( c_{x}^\dagger c_{x'} - \text{h.c.} \right) \]

Kubo

\[ \sigma = \frac{\langle JJJ \rangle}{\omega} \]

\[ \sigma \sim \frac{1}{\omega} \int d\tau \ t^2 \left\langle c_{x}^\dagger c_{x'} c_{x}^\dagger c_{x} \right\rangle \sim \frac{t^2}{E^2} G(\tau)^2 \sim \frac{t^2}{E^2} \left( \frac{1}{\sqrt{U\tau}} \right)^2 \]

\[ \sim \frac{t^2}{U} \frac{1}{k_{B}T} \]
Wiedemann-Franz ratio

Lorenz

\[ L = \frac{\kappa}{\sigma T} = \frac{\pi^2}{3} \text{ for a Fermi liquid} \]

SYK lattice:

\[ L = \mathcal{L}(T/E_c) \]

YbRh\textsubscript{2}Si\textsubscript{2}, Pfau et al (2012)
Kadowaki Woods ratio

\[ \rho_\zeta(T \ll E_c) \approx \rho_\zeta(0) + A_\zeta T^2, \]

\[ KW = A/\gamma^2 \quad \text{approximately constant for many metals} \]

Scaling implies:

\[ A \sim 1/(NE_c^2) \]

recall \[ \gamma \sim 1/E_c \]

\[ KW = A/(N\gamma)^2 \sim 1/N^3 \quad \text{independent of } t,U! \]
SYK metal

- Small coherence scale $E_c = t^2/U$
- Heavy mass $\gamma \sim m^*/m \sim U/t$
- Small QP weight $Z \sim t/U$
- Kadowaki-Woods $A/\gamma^2 = \text{constant}$
- Linear in $T$ resistivity and $T/\kappa$
- Lorenz ratio crosses over from FL to NFL value
SYK Fermi Surfaces?

- Extension to translationally invariant systems?

![Fermi surface emerging in translationally invariant SYK model by Aavishkar Patel](image)

\[
G(k, i\omega_n)^{-1} = i\omega_n - \epsilon(k) - \Sigma(k, i\omega_n),
\]
\[
\Sigma(x, \tau) = -U_0^2 G(x, \tau)^2 G(-x, -\tau),
\]

- SYK lattice, tensor models, ...
- Momentum space differentiation and realistic applications?
- Relation to methods like DCA, cluster DMFT?
SYK Fermi Surfaces?


SYK dots as strong local scatterers

\[ H = -t \sum_{\langle rr' \rangle: \ i=1}^{M} (c_{ri}^\dagger c_{r'i} + \text{h.c.}) - \mu_c \sum_{r; \ i=1}^{M} c_{ri}^\dagger c_{ri} - \mu \sum_{r; \ i=1}^{N} f_{ri}^\dagger f_{ri} \]
\[ + \frac{1}{NM^{1/2}} \sum_{r; \ i,j=1}^{N} \sum_{k,l=1}^{M} g_{ijkl} f_{ri}^\dagger f_{rj}^\dagger c_{rk} c_{rl} + \frac{1}{N^{3/2}} \sum_{r; \ i,j,k,l=1}^{N} J_{ijkl} f_{ri}^\dagger f_{rj}^\dagger f_{rk} f_{rl}. \]

Chowdhury, Werman, Berg, Senthil, arXiv:1801.06178

SYK models w/ full translational symmetry

\[ H_c = \sum_{r,r'} \sum_{\ell} (-t_{r,r'}^c - \mu_c \delta_{r,r'}) c_{r\ell}^\dagger c_{r'\ell} + \frac{1}{(2N)^{3/2}} \sum_r \sum_{ijkl} U_{ijkl}^c c_{ri}^\dagger c_{rj}^\dagger c_{rk} c_{rl}, \]
\[ H = -t \sum_{\langle rr' \rangle: \ i=1}^{M} (c_{ri}^\dagger c_{r'i} + \text{h.c.}) - \mu_c \sum_{r; \ i=1}^{M} c_{ri}^\dagger c_{ri} - \mu \sum_{r; \ i=1}^{N} f_{ri}^\dagger f_{ri} \]
\[ + \frac{1}{NM^{1/2}} \sum_{r; \ i,j=1}^{N} \sum_{k,l=1}^{M} g_{ijkl} f_{ri}^\dagger f_{rj}^\dagger c_{rk} c_{rl} + \frac{1}{N^{3/2}} \sum_{r; \ i,j,k,l=1}^{N} J_{ijkl} f_{ri}^\dagger f_{rj}^\dagger f_{rk} f_{rl}. \]
Quantum magnets

Less obvious ultra-quantum "fluids"
A growing experimental effort valuable for its sensitivity, long-wavelength nature, and automatic removal of localized states

Heat transport in magnets

M. Yamashita et al, 2010

Tb$_2$Ti$_2$O$_7$
Ong’s group 2015

Pr$_2$Zr$_2$O$_7$
Tokiwa, Matsuda et al. (2017?)
SYK magnets?

Original SY paper actually treated spins

Challenge: extend to non-disordered systems

• Similar to problem of SYK Fermi surfaces
• Tensor model for large N control?

\[
\begin{align*}
\mathcal{H} &= \frac{1}{\sqrt{NM}} \sum_{i>j} J_{ij} \hat{S}_i \cdot \hat{S}_j \\
\sum = & \frac{J}{N^{3/2}} \sum_{\langle ij \rangle} S_{i;abcd} S_{j;adcb} \\
S_{i;abcd} &= f_{i;Aab} f_{j;Acd}
\end{align*}
\]
Conclusion

• Convergence of ideas suggests that maybe problematic metals are not “strange”, “bad”, or “incoherent” but rather they are SYK